The UNNS Path Integral Protocol (UPIP): Sum-Over-Histories for Recursive Systems

UNNS Research Notes

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Abstract

We introduce the *UNNS Path Integral Protocol* (UPIP), extending the Quantization Protocol (UQP) into a full sum-over-histories formulation. Recursive sequences are treated as trajectories through nest space. Each trajectory contributes with a phase weight given by a recursive action functional. This formalism parallels Feynman's path integral and opens a statistical and quantum view of recursion.

Contents

1 Recursive Trajectories

Definition 1.1 (Recursive Trajectory). *A recursive trajectory* γ *is a sequence of nest states*

$$\gamma = (\mathcal{N}_0 \to \mathcal{N}_1 \to \cdots \to \mathcal{N}_T),$$

generated by recursive update rules

$$\mathcal{N}_{t+1} = f(\mathcal{N}_t, a_t).$$

Remark 1.2. *Each trajectory is a "history" of recursion, analogous to a path in mechanics.*

2 Recursive Action Functional

Definition 2.1 (Recursive Action)**.** *Given a recursion with coefficients a_t, define the action*

$$S[\gamma] = \sum_{t=0}^{T-1} L(\mathcal{N}_t, \mathcal{N}_{t+1}, a_t),$$

where L is a Lagrangian encoding recursive cost.

For linear recursions $\mathcal{N}_{t+1} = a\mathcal{N}_t$, take

$$L(\mathcal{N}_t, \mathcal{N}_{t+1}, a) = \frac{1}{2}\big(\mathcal{N}_{t+1} - a\mathcal{N}_t\big)^2.$$

3 Path Integral Formulation

Definition 3.1 (Partition Function)**.** *The UNNS path integral is*

$$Z = \sum_{\gamma} e^{iS[\gamma]},$$

summing over all admissible recursive trajectories γ.

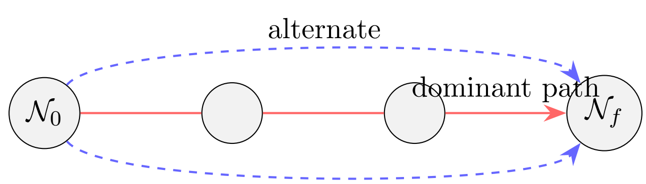
Definition 3.2 (Propagator)**.** *The amplitude to evolve from $\mathcal{N}_i$ to $\mathcal{N}_f$ in time T is*

$$K(\mathcal{N}_f, T; \mathcal{N}_i, 0) = \sum_{\gamma:\mathcal{N}_i \to \mathcal{N}_f} e^{iS[\gamma]}.$$

Theorem 3.3 (Stationary Phase Principle)**.** *Dominant contributions to K arise from trajectories minimizing $S[\gamma]$, yielding recursive Euler–Lagrange equations*

$$\frac{\partial L}{\partial \mathcal{N}_t} - \frac{d}{dt}\Big(\frac{\partial L}{\partial \dot{\mathcal{N}}_t}\Big) = 0.$$

4 Diagrammatic Overview



5 Applications

5.1 Mathematics

- Defines recursive analogs of least-action principles.
- Provides generating functions via partition sums.

5.2 Physics

- Models recursion as discrete quantum field evolution.
- Suggests UNNS analogues of propagators and Green's functions.

5.3 Computation

- Enables probabilistic simulation of recursion.
- Suggests quantum-inspired algorithms for recursive search.

6 Conclusion

The UNNS Path Integral Protocol frames recursion as a sum-over-histories. Trajectories contribute phase-weighted amplitudes, and dominant histories satisfy recursive Euler–Lagrange principles. This positions UNNS as a bridge between recursion, physics, and computation.